

Lecture Notes on Section 3.2

Logistic Growth Model

Or

Limited Growth Model

Objective

- To study a standard model of population growth in a constrained environment.
- To explore various aspects of logistic population growth models, such as growth rate and carrying capacity.
- To understand discrete and continuous growth models using mathematically defined equations.

Logistic Growth

Most populations cannot continue to grow forever because they will eventually run out of water, food, sunlight, space or other resources. As these resources begin to run out, population growth will start to slow down. When the growth rate of a population decreases as the number of individuals increases is the logistic population growth

The study of the population growth, when populations are subjected to constraints like limited food supply, etc. while considering the existence of intraspecific competition (competition between the individuals of the same species) is termed as Logistic Growth.

Carrying Capacity: The largest population that can be supported indefinitely within the resources available in the environment

Logistic growth assumes that systems grow exponentially until an upper limit or "carrying capacity" inherent in the system approaches, at which point the growth rate slows and eventually saturates, producing the characteristic S-shape curve (Stone, 1980). Logistic models are density dependent where the growth rate is equal to the birth rate minus the death rate.

Equation of Logistic Model

$$\frac{dX}{dt} = r X \left(1 - \frac{X}{K} \right)$$

$X(t)$ Is the "Population Size" at time t , r is growth rate & K is carrying capacity. We consider $r > 0$ & $K > 0$

It is a nonlinear differential equation.

$$K = r/\gamma$$

(Ref. density dependence growth)

Carrying Capacity: The carrying capacity is the maximum number of individuals of a given species that an area's resources can sustain indefinitely without significantly depleting or degrading those resources.

The factor $(1 - X/K)$ in the Logistic Equation determines how close is the Population Size to the Limit K , which means as the population gets closer and closer to the limit, the growth gets slower and slower.

“It explains how density dependent limiting factors eventually decrease the growth rate until a population reaches a Carrying Capacity (K).”

Comparing Exponential & Logistic Growth

Exponential Growth equation:

$$\frac{dX}{dt} = r X, X(t)$$

is the population size at time t, r is the growth rate

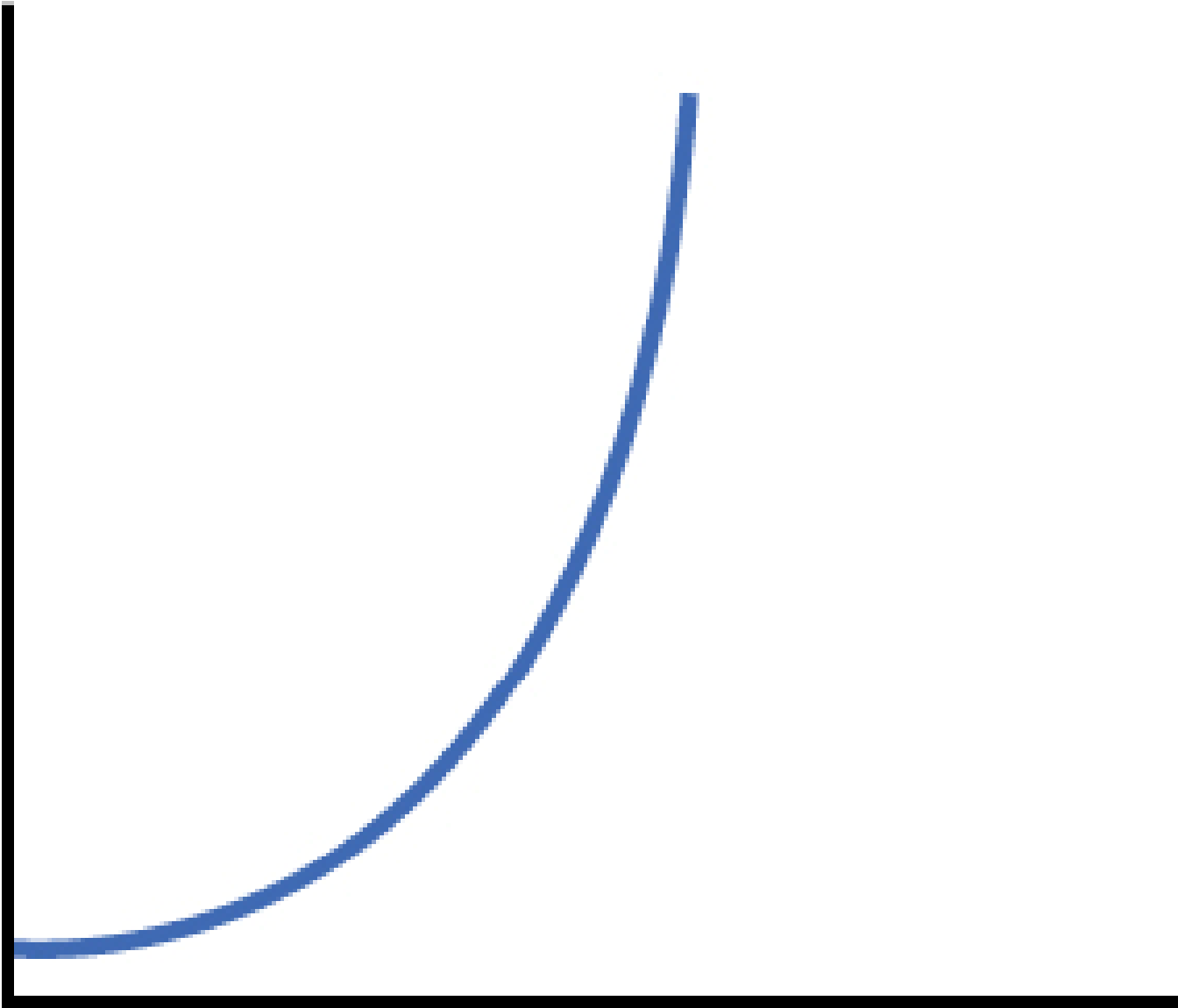
In **exponential growth**, a population's *per capita* (per individual) growth rate r stays the same regardless of population size, making the population grow faster and faster as it gets larger. In nature, populations may grow exponentially for some period, but they will ultimately be limited by resource availability.

In **logistic growth**, a population's *per capita* growth rate gets smaller and smaller as population size approaches a maximum imposed by limited resources in the environment, known as the **carrying capacity** (K).

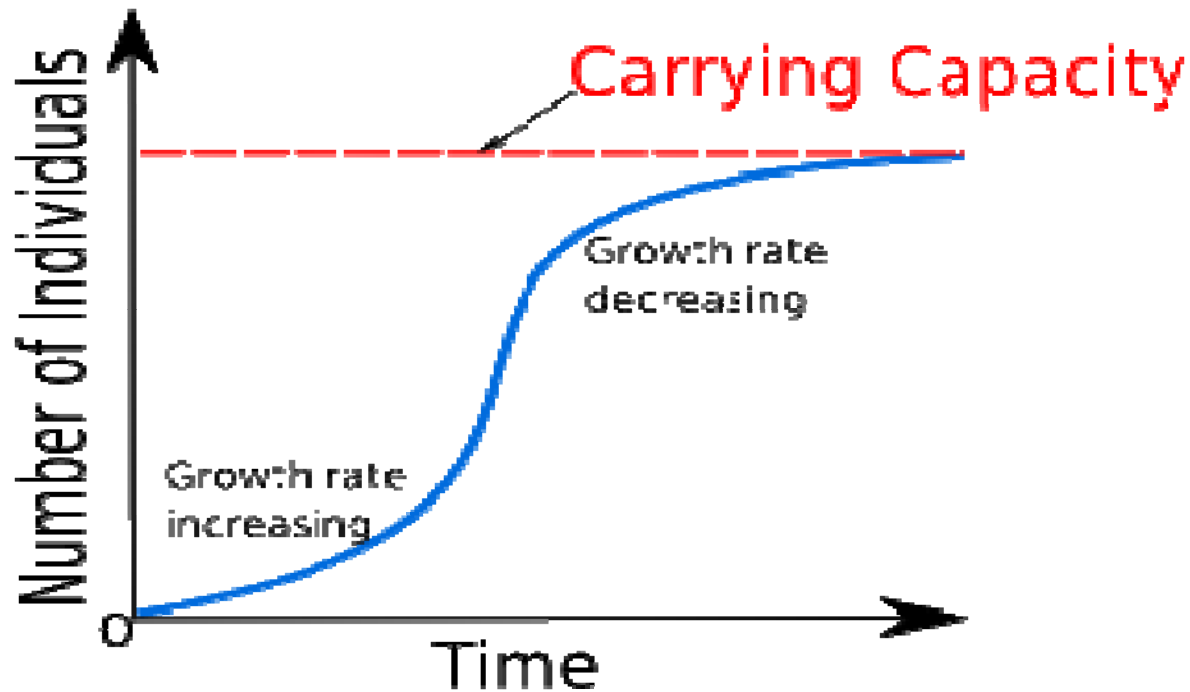
Exponential growth produces a **J-shaped curve**, while logistic growth produces an **S-shaped curve**.

Exponential Growth

Population size



Logistic Equation Graph



“Fig. explains how density dependent limiting factors eventually decrease the growth rate until a population reaches a Carrying Capacity (K).”