Dear Students

Hope you all doing well.

As all of us are aware that the pandemic caused by novel Corona virus has given the entire world a devastating blow. Our nation is no exception here. I request you all to maintain the social distancing and personal hygiene which are the only keys to prevent COVID 19 infections. Therefore, direct class to class learning is not possible. Hopefully in our last class, we have covered almost 85% of our syllabus. I request you all that to go through following notes, link or references just to finish the remaining portion.

Apart from this, all of you can contact me through E-mail, Whatsapp or Mobile for any quarry related to our course of Wave and Optics. In our next lecture, we will try to do some numerical problems and your queries related to this lecture.

Thanking you.

With Best Wishes

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Reference Books:

- Vibrations and Waves, A.P. French, 1st Edn. 2003, CRC press.
- The Physics of Waves and Oscillations, N.K. Bajaj, 1998, Tata McGraw Hill.
- Fundamentals of Optics, F.A Jenkins and H.E White, 1976, McGraw-Hill
- Principles of Optics, B.K. Mathur, 1995, Gopal Printing
- Fundamentals of Optics, A. Kumar, H.R. Gulati and D.R. Khanna, 2011, R. Chand Publications
- University Physics. F.W. Sears, M.W. Zemansky and H.D. Young. 13/e, 1986. Addison-Wesley
- Optics, Ajoy Ghatak, 2008, Tata McGraw Hill

Some important Link

https://ocw.mit.edu/courses/mechanical-engineering/2-71-optics-spring-2014/lecture-notes/

https://www.physicsbyfiziks.com/freedownload/sample-material/

Group Velocity and Phase Velocity

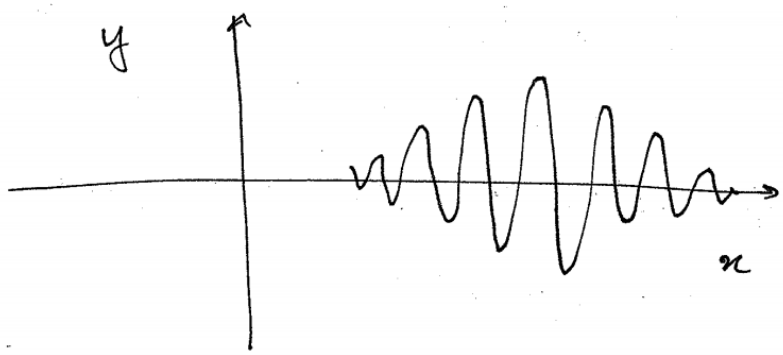
- Consider 2 plane waves:

$$y_1(x,t) = A\cos[(\omega+\Delta\omega)t - (k+\Delta k)x]$$

$$y_2(x,t) = A\cos[(\omega-\Delta\omega)t - (k-\Delta k)x]$$

Superposition gives:

$$y(x,t) = y_1 + y_2 = 2A\cos(\omega t - kx)\cos((\Delta\omega)t - (\Delta k)x)$$



$$\boxed{v_p = \frac{\omega}{k}}$$

But entire packet as a whole moves towards right with vel. $\boxed{\frac{\Delta\omega}{\Delta k} = v_g}$

- Consider case when there are an infinite no. of waves with continuously varying frequencies.

$$y(x,t) = \int_{-\infty}^{\infty} A(\omega)\, e^{i(\omega t - kx)}\, d\omega$$

$$y(0,t) = \int_{-\infty}^{\infty} A(\omega)\, e^{i\omega t}\, d\omega$$

Replace to get $y(x,t)$

$$\Rightarrow \quad A(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} y(0,t)\, e^{-i\omega t}\, dt$$

(i) Non dispersive medium : (Take example of light)

$$k(\omega) = \frac{\omega}{c}$$

$$\cancel{y(x,t) =} \; E(z,t) = \int_{-\infty}^{\infty} A(\omega)\, e^{-i\frac{\omega}{c}(z - ct)}\, d\omega$$

* $\therefore$ It is a fn. of $(z-ct)$. Hence pulse propagates with vel. c

(ii) Dispersive medium :

$$k(\omega) = \frac{\omega}{c} n(\omega)$$

$$E(z,t) = \int_{-\infty}^{\infty} A(\omega)\, e^{i(\omega t - k(\omega) z)}\, d\omega$$

A is generally significant over a small region say $\omega_0 - \Delta\omega$ & $\omega_0 + \Delta\omega$

$$\therefore \quad E(z,t) = \int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} A(\omega)\, e^{i(\omega t - kz)}\, d\omega$$

Using taylor series about ω_0

$$k(\omega) = k(\omega_0) + (\omega - \omega_0) \underbrace{\left.\frac{dk}{d\omega}\right|_{\omega=\omega_0}}_{\frac{1}{V_g}} + \cdots$$

$$\Rightarrow \quad E(z,t) = \int_{-\infty}^{\infty} A(\omega)\, e^{-i\left(k_0 z + \frac{\omega-\omega_0}{V_g} z - \omega t\right)}\, d\omega$$

Where $k_0 = k(\omega_0)$ & $\frac{dk}{d\omega}$ has been written as $\frac{1}{V_g}$. Integration limits can be extended since ~~for~~ $A(\omega)$ is negligible outside limits.

Writing ωt as $(\omega - \omega_0)t + \omega_0 t$

$$E(z,t) = \underbrace{e^{i(\omega_0 t - k_0 z)}}_{\text{Phase term}} \underbrace{\int_{-\infty}^{\infty} A(\Omega)\, e^{-i\frac{\Omega}{V_g}(z - V_g t)}\, d\Omega}_{\text{Envelope}}$$

where $\Omega = \omega - \omega_0$

Clearly Envelope travels at vel. $\boxed{V_g = \frac{d\omega}{dk}}$

- Reln b/w V_p & V_g

$$V_g = \frac{d\omega}{dk} = \frac{d}{dk}(kV_p)$$

$$\Rightarrow \boxed{V_g = V_p + k\frac{dV_p}{dk}}$$

Now $k = \frac{2\pi}{\lambda}$

$$\Rightarrow V_g = V_p + k\frac{dV_p}{d\lambda}\frac{d\lambda}{dk}$$

$$= V_p + \frac{2\pi}{\lambda} \times \left(-\frac{2\pi}{k^2}\right)\frac{dV_p}{d\lambda}$$

$$\Rightarrow \boxed{V_g = V_p - \lambda\frac{dV_p}{d\lambda}}$$

Using $V_p = \frac{c}{n}$

$$V_g = V_p + \lambda\frac{c}{n^2}\frac{dn}{d\lambda}$$

$$\boxed{V_g = V_p\left(1 + \frac{\lambda}{n}\frac{dn}{d\lambda}\right)}$$

Clearly $\frac{dn}{d\lambda} = 0$ (Non dispersive medium) $\Rightarrow$

$V_g = V_p$ (as obtained earlier)

Superposition of waves & Beats

- Consider two plane waves:

$$y_1 = a_1 \sin(\omega t - kx)$$

$$y_2 = a_2 \sin(\omega t - kx + \varphi)$$

$$y = y_1 + y_2 =$$

$$a_1 \sin(\omega t - kx) + a_2 \sin(\omega t - kx)\cos\varphi + a_2 \sin\varphi \cos(\omega t - kx)$$

$$= [a_1 + a_2\cos\varphi]\sin(\omega t - kx) + [a_2 \sin\varphi]\cos(\omega t - kx)$$

$$= (A\cos\delta)\sin(\omega t - kx) + (A\sin\delta)\cos(\omega t - kx)$$

where $A\cos\delta = a_1 + a_2\cos\varphi$

$A\sin\delta = a_2 \sin\varphi$

$$\therefore \quad A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\varphi}$$

$$\delta = \tan^{-1}\left(\frac{a_2\sin\varphi}{a_1 + a_2\cos\varphi}\right)$$

$$\Rightarrow \quad y = A\sin(\omega t - kx + \delta)$$

$A(\varphi)$ is max when $\varphi = 2n\pi$

$A(2n\pi) = a_1 + a_2$

Path diff $= \frac{\lambda}{2\pi} \times 2n\pi = n\lambda$

$n = 0, 1, 2, ---$

Constructive Interference.

Also, $I = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

- For destructive interference: $\phi = (2n+1)\pi$

$$A_{min} = A((2n+1)\pi) = |a_1 - a_2|$$

$$\text{Path diff.} = \frac{\lambda}{2\cancel{\pi}} \times (2n+1)\cancel{\pi} = \left(n+\frac{1}{2}\right)\lambda$$

$$n = 0, 1, 2, \text{---}$$

Beats

Consider superposition of 2 waves with slightly diff. frequencies at a pt. in space.

$$y_1 = a \sin \omega_1 t$$
$$y_2 = a \sin \omega_2 t.$$

$$y = y_1 + y_2 = \underbrace{2a \cos\left(\frac{\omega_1 - \omega_2}{2}\right)t}_{\text{Amplitude varies}} \; \underbrace{\sin\left(\frac{\omega_1 + \omega_2}{2}\right)t}_{\text{Avg. frequency heard.}}$$

$\therefore$ Beats is a phenomenon of interference of two waves slightly diff. in frequency where avg. frequency is heard & intensity oscillates periodically.

$$I \propto A^2$$
$$= \cos^2\left(\frac{\omega_1 - \omega_2}{2}\right)t$$

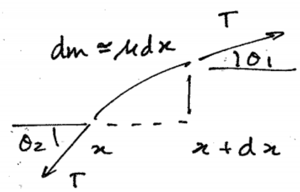
Max intensity when $\left(\frac{\omega_1 - \omega_2}{2}\right)t = n\pi$

$\therefore$ Time b/w 2 maximas $= \dfrac{2\pi}{\omega_1 - \omega_2}$

or $\boxed{\omega_{beat} = \omega_1 - \omega_2}$ Beat frequency.

Standing Waves

1) Transverse wave on string wave eq^n. :



eq^n. of motion is $(\mu dx) \dfrac{\partial^2 y}{\partial t^2} = T[\sin\theta_1 - \sin\theta_2]$

$\sin\theta \approx \tan\theta = \dfrac{\partial y}{\partial x}$

$$\Rightarrow (\mu dx) \frac{\partial^2 y}{\partial t^2} = T\left[\left.\frac{\partial y}{\partial x}\right|_{x+dx} - \left.\frac{\partial y}{\partial x}\right|_{x} \right]$$

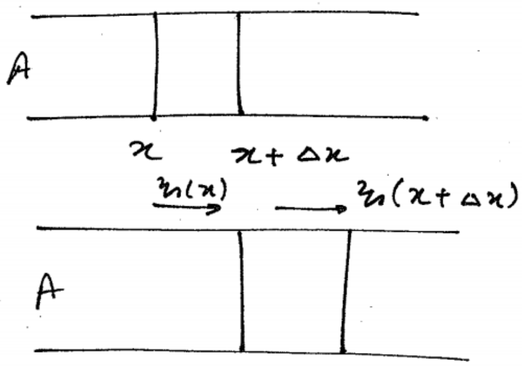
$$= T \frac{\partial^2 y}{\partial x^2} dx$$

$$\Rightarrow \frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu} \frac{\partial^2 y}{\partial x^2}$$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T} \frac{\partial^2 y}{\partial t^2}$$

$$\Rightarrow v = \sqrt{\frac{T}{\mu}}$$

2) Longitudinal wave in gases -



Total force towards left is :

$$[\Delta P(x) - \Delta P(x+\Delta x)]A$$ where ΔP is change in pressure

$$\Rightarrow \cancel{f} \; -\frac{\partial}{\partial x}(\Delta P)A\Delta x = \rho A\Delta x \frac{\partial^2 \xi_1}{\partial t^2}$$

$$\Rightarrow -\frac{\partial}{\partial x}(\Delta P) = \rho\frac{\partial^2 \xi_1}{\partial t^2}$$

~~Now assuming adiabatic~~

$$\Delta P = -B\left(\frac{\Delta V}{V}\right)$$

Change in length $= [\xi_1(x+\Delta x) - \xi_1(x) + \Delta x] - \Delta x$

$$= \frac{\partial \xi_1}{\partial x}\Delta x$$

$$\therefore \quad \Delta V = \frac{\partial \xi_1}{\partial x}A\Delta x$$

$$\Rightarrow \boxed{\Delta P = -\frac{B}{\cancel{V}}\left(\frac{\partial \xi}{\partial x}\right) \cancel{A\Delta x}} \rightarrow \text{Pressure wave}$$

$$\Rightarrow \frac{\partial^2 \xi}{\partial x^2} = \frac{\rho}{B}\frac{\partial^2 \xi}{\partial t^2}$$

$$\Rightarrow v = \sqrt{\frac{B}{\rho}}$$

Energy in wave motion

$$\Delta P = -B\frac{\partial \xi}{\partial x} = \frac{F}{A} = f$$

~~Inter~~ Power per unit area $= f v = f\left(\frac{\partial \xi}{\partial t}\right)$

$\therefore$ Power per unit area will be :

$$\xi = a \sin(\omega t - kx)$$

$$\frac{\partial \xi}{\partial x} = -ak\cos(\omega t - kx)$$

$$\frac{\partial \xi}{\partial t} = a\omega\cos(\omega t - kx)$$

$$\Rightarrow B a^2 k \omega \cos^2(\omega t - kx)$$

$$I = \langle P \rangle_{avg} = \frac{1}{2} B a^2 k \omega$$

Using $B/\rho = v^2$ & $kv = \omega$

$$I = \frac{1}{2}\rho v a^2 \omega^2 = \boxed{2\pi^2 \rho v a^2 \nu^2}$$

For transverse, we'll have μ instead of ρ.

Standing waves

- Consider 2 plane waves with same amplitude, frequence but opposite direction.

$$y_1 = a\sin(\omega t - kx)$$

$$y_2 = a\sin(\omega t + kx)$$

↳ reflected wave in phase if end is not fixed.

$$y = y_1 + y_2 = (2a\cos kx)\sin\omega t$$

↳ Amplitude.

- ∴ All particles oscillate ~~to~~ in phase but amplitude is different.

No $(kx - \omega t)$ term $\Rightarrow v = 0$

- **Nodes** = Points where amplitude is 0 at all times.

$$\Rightarrow \cos kx = 0$$

$$\Rightarrow kx = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow x = (2n+1)\frac{\lambda}{4}.$$

∴ Nodes occur after $\frac{\lambda}{2}$ distance: (Similarly antinodes)

- Now if both ends of a string are fixed, Reflected wave should be 'π' out of phase so that $y = 0 \ \forall t$ at $x = 0$

$$\Rightarrow \quad y_1 = a \sin(\omega t - kx)$$

$$y_2 = -a \sin(\omega t + kx)$$

$$= \cancel{2a\cos} -2a \sin kx \cos \omega t$$

$\therefore \quad kx = n\pi$ for nodes

or $\boxed{x = \frac{n\lambda}{2}}$

Also, ~~for~~ $x = L$ is a node

$$\therefore \quad \frac{2L}{n} = \lambda$$

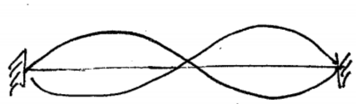
$\Rightarrow$ Only $\nu = \frac{nv}{2L}$ are allowed.

$n = 1$ is fundamental freq. or 1st harmonic & so on. These are called normal modes or natural frequencies.

- Similarly antinodes will be at :

$$\boxed{x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}}$$

1st harmonic

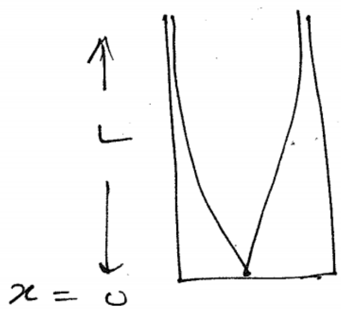


2nd harmonic.

- Energy transmission: Since $v = 0$, clearly from the expression for I, there is no transmission of energy.

Standing waves in pipes (Sound waves)

1) Closed on one end.



~~Eqn. will be $(2a\cos kx)\sin\omega t$~~

L is ~~an antinode~~ ~~a node~~ an antinode.

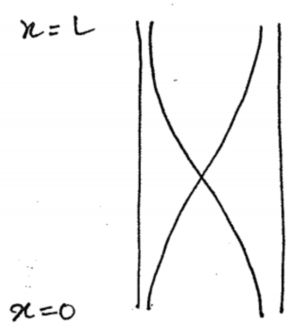
$$\Rightarrow x = \left(n+\frac{1}{2}\right)\frac{\lambda}{2}$$

$$\Rightarrow \lambda = \frac{2L}{n+1/2}$$

$$\nu = \left(n+\frac{1}{2}\right)\frac{v}{2L}, \quad n = 0, 1, \ldots$$

∴ ~~freq.~~ Allowed frequencies are odd multiples of $\frac{v}{4L}$ i.e. $\frac{v}{4L}$, $3\frac{v}{4L}$, $5\frac{v}{4L}$ (Odd harmonics)

2) Open on both sides.



$$y_1 = a\sin(\omega t - kx)$$

$$y_2 = a\sin(\omega t + kx) \text{ (No phase change)}$$

$$\Rightarrow y = 2a\cos kx \sin\omega t$$

$$kL = n\pi$$

$$\Rightarrow \lambda = \frac{2L}{n}$$

$$\nu = \frac{n}{2L}v \text{ (All harmonics)}$$

Electromagnetic wave and its Transverse Nature

An electromagnetic wave in a vacuum consists of mutually perpendicular and oscillating electric and magnetic fields. The wave is a transverse wave, since the fields are perpendicular to the direction in which the wave travels. All electromagnetic waves, regardless of their frequency, travel through a vacuum at the same speed, the speed of light c ($c = 3.00 \times 10^{8}$ m/s).

The frequency f and wavelength λ (lambda) of an electromagnetic wave in a vacuum are related to its speed through the relation

$$c = f\lambda .$$

The wave speed of an electromagnetic wave in a dielectric medium is given by $v = 1/(\varepsilon_0 \mu_0)^{1/2}$, where ε_0 and μ_0 are the permittivity and permeability of the dielectric respectively. Electromagnetic waves cannot propagate within a conductor; they are totally reflected when they strike a conducting surface.

∴ To show electromagnetic wave transverse we have to prove $\vec{E}$, $\vec{B}$ and $\vec{K}$ are ~~per~~ mutually perpendicular to each other.

As we know that

$$\vec{E} = E_0 e^{i(\vec{K}\cdot\vec{r} - \omega t)} \quad \text{---} \quad (9)$$

$$\vec{H} = \vec{H}_0 e^{i(\vec{K}\cdot\vec{r} - \omega t)} \quad \text{---} \quad (10)$$

taking divergence of eq^n (9)

$$\vec{\nabla}\cdot\vec{E} = \vec{\nabla}\cdot\left\{\vec{E}_0 e^{i(\vec{K}\cdot\vec{r} - \omega t)}\right.$$

Here

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\vec{K} = K_x\hat{i} + K_y\hat{j} + K_z\hat{k}$$

$$\vec{r} = x\cdot\hat{i} + y\cdot\hat{j} + z\cdot\hat{k}$$

$$\vec{E_0} = E_{0x}\cdot\hat{i} + E_{0y}\cdot\hat{j} + E_{0z}\cdot\hat{k}$$

$$\vec{\nabla}\cdot\vec{E} = \left\{\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right\}\cdot\left\{E_0 e^{i(\bar{K}\cdot\bar{r}-\omega t)}\right\}$$

$$\left\{\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right\}\left\{(E_{0x}\hat{i} + E_{0y}\hat{j} + E_{0z}\hat{k})\, e^{i(xK_x + yK_y + zK_z - \omega t)}\right\}$$

$$\Rightarrow \frac{\partial}{\partial x}\left\{E_{0x} e^{i(\bar{K}\cdot\bar{r}-\omega t)}\right\} + \frac{\partial}{\partial y}\left\{E_{0y} e^{i(\bar{K}\cdot\bar{r}-\omega t)}\right\}$$

$$+ \frac{\partial}{\partial z}\left\{E_{0z} e^{i(\bar{K}\cdot\bar{r}-\omega t)}\right\}$$

On differentiating above eqn

$$\Rightarrow \left\{E_{0x} e^{i(\bar{K}\cdot\bar{r}-\omega t)}\, i\cdot K_x\right\} + \left\{E_{0y} e^{i(\bar{K}\cdot\bar{r}-\omega t)}\, i\, K_y\right\}$$

$$\left\{+ E_{0z} e^{i(\bar{K}\cdot\bar{r}-\omega t)}\, iK_z\right\}$$

$$\Rightarrow \vec{\nabla}\cdot\bar{E} = i e^{i(\bar{K}\cdot\bar{r}-\omega t)}\left[K_x\cdot E_{0x} + K_y E_{0y} + K_z E_{0z}\right]$$

$$\Rightarrow \vec{\nabla}\cdot\vec{E} = i e^{i(\bar{K}\cdot\bar{r}-\omega t)}\, \vec{K}\cdot\vec{E_0}$$

$\vec{\nabla} \cdot \vec{E} = i\bar{K} \cdot \bar{E}$

Now from maxwell's first eq^n in free space. we have -:

$\vec{\nabla} \cdot \vec{E} = 0$

Thus ~~ø~~

$0 = ik \cdot \vec{E}$

$\Rightarrow$ ~~E = E~~ $\boxed{\vec{K} \cdot \vec{E} = 0}$

$\Rightarrow$ ~~Now~~ K perperpendicular to $\vec{E}$

Now again taking the div. of eq^n (13) we have.

$$\bar{\nabla} \cdot \bar{H} = \bar{\nabla} \cdot \left\{ \vec{H}_0 \, e^{i(\bar{K} \cdot \bar{r} - \omega t)} \right\}$$

on solving above eq^n we get

$\bar{\nabla} \cdot \bar{H} = i\bar{K} \cdot \bar{H}$

from maxwell's second eq^n in free space we get -:

$\bar{\nabla} \cdot \bar{H} = 0$

Thus

Now we have to show $-\ \vec{E} \perp \vec{H}$

From Maxwell's third eq^n

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}, \quad \vec{B} = \mu_0 \vec{H}$$

$$i\vec{K} \times \vec{E} = -\mu_0 \frac{\partial}{\partial t} \left\{ \vec{H}_0 e^{i(\vec{K}\cdot\vec{r} - \omega t)} \right\}$$

$$i\vec{K} \times \vec{E} = -\mu_0 \vec{H}_0 \frac{\partial}{\partial t} \left\{ e^{i(\vec{K}\cdot\vec{r} - \omega t)} \right\}$$

$$i\vec{K} \times \vec{E} = -\mu_0 \vec{H}_0 e^{i(\vec{K}\cdot\vec{r} - \omega t)} \cdot (-i\omega)$$

$$i\vec{K} \wedge \vec{E} = i\mu_0 \omega H_0 e^{i(\vec{K}\cdot\vec{r} - \omega t)}$$

$$\boxed{\vec{K} \times \vec{E} = \mu_0 \omega \vec{H}}$$

from above relation it is clear that $H \perp E$. Similarly using maxwell's fourth equation we can prove $\vec{E} \perp \vec{H}$

Thus including all above results we can say that $\vec{E}$, $\vec{H}$ and $\vec{K}$ are