The Twin Paradox

The Twin Paradox is a relativistic effect that shows up when different observers take different paths between two events in spacetime and find that they experience different amounts of time. The classic narrative for describing the Twin Paradox, not surprisingly, involves twins and, somewhat surprisingly, doesn't involve paradoxes. The story is this: One starts with twins on Earth, A and B, who are clearly the same age. B leaves Earth (event 1) for a trip to another star at nearly the speed of light, then turns around and comes back. When the trip is over and the twins are reunited (event 2), A will literally be older than B. The traveling twin experiences less time. Why?

To understand this, we need to do a little calculation. Suppose the difference between A and B's velocity is 0.9c. A sits still (from his perspective) while B goes to a star system 9 light years far in 10 years at 0.9c and travels back for another 10 years. Thus, from A's perspective reunion (event 2) takes place after 20 years.

Here we need the appropriate measure for spacetime distance called the the "Lorentz Interval": $\Delta S^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$. The advantage here is that the interval between any two "events" in spacetime is always the same (invariant), despite any rotating, shifting, or relativistic weirdnesses. Using this space-time interval we can figure out how much time they each experience. A experiences 20 years ($20^2 - 0^2 = 20^2$) while B experiences two 4.36 year trips ($10^2 - 9^2 = 4.36^2$), i.e. 8.72 years. Thus, the Twin Paradox is due to the weird way geometry works in spacetime vs. space. In normal geometry a straight line is the shortest path between two points, and that distance is measured with a ruler. But in spacetime a straight line is the longest path between two events, and that "distance" is measured with a clock. In a nutshell: the more circuitous a path is, the less time will be experienced by objects following that path. The twin paradox has nothing to do with who's moving and who's stationary.

Let us solve the above problem in a different way: As per A, the round-trip takes 20 years. So, for A, including himself, everybody on Earth will be 20 years older when B returns. The amount of time as measured on B's clocks and the aging of B during his trip will be reduced by the reciprocal of the Lorentz factor (time dilation) $\varepsilon = \sqrt{(1 - \frac{v^2}{c^2})}$. In this case, it is 0.436. So, B will have aged only $0.436 \times 20 = 8.72$ years when he returns. B also calculates the particulars of the trip from his own perspective. He knows that the distant star and the Earth are moving relative to the ship at speed $v = 0.9c$ during the trip. In their rest frame the distance between the Earth and the star system is εd where $d = 9$ light years. Thus, $\varepsilon d = 0.436 \times 9 = 3.924$ light years (length contraction), for both the forward and the return journeys. Each half of the journey takes $\varepsilon d / v = 3.924 / 0.9 = 4.36$ years, and the round trip takes twice as long (8.72 years). His calculations show that he will arrive home having aged 8.72 years. Thus B's calculation about his aging is in complete agreement with the preceding calculations.