Que. 1 Identify the co-ordinates of any local & absolute exteremes points and inflection points. Graph of the function.

(i) $y = x^4 + 2x^3$ (ii) $y = x^4 - 2x^2$.

Solution. (i) $y = x^4 + 2x^3$

then $y' = 4x^3 + 6x^2 = 2x^2(2x+3)$ —— (1)

and $y'' = 12x^2 + 12x = 12x(x+1)$. —— (2)

Thus the Critical points of f occur only at the roots of y'.

i.e. $y' = 0$

$\Rightarrow 2x^2(2x+3) = 0$

$\Rightarrow$ either $x = 0$ or $x = -\frac{3}{2}$.

Critical points.

$x = 0$ and $x = -\frac{3}{2}$

Intervals	$x < -\frac{3}{2}$	$-\frac{3}{2} < x < 0$	$x > 0$.
Singn of y'	$-$	$+$	$+$
Behavior of y	decreasing ($\downarrow$)	Increasing ($\uparrow$)	$\uparrow$

at $x = 0$, $y'' = 0$

no extermum at $x = 0$

at $x = -\frac{3}{2}$, y'' is positive.

So there is a local minimum at $x = -\frac{3}{2}$.

Using above table. we see that y is $\downarrow$ on $(-\infty, -\frac{3}{2})$ and $\uparrow$ (increasing) on $(-\frac{3}{2}, \infty)$.

Now $y'' = 0$

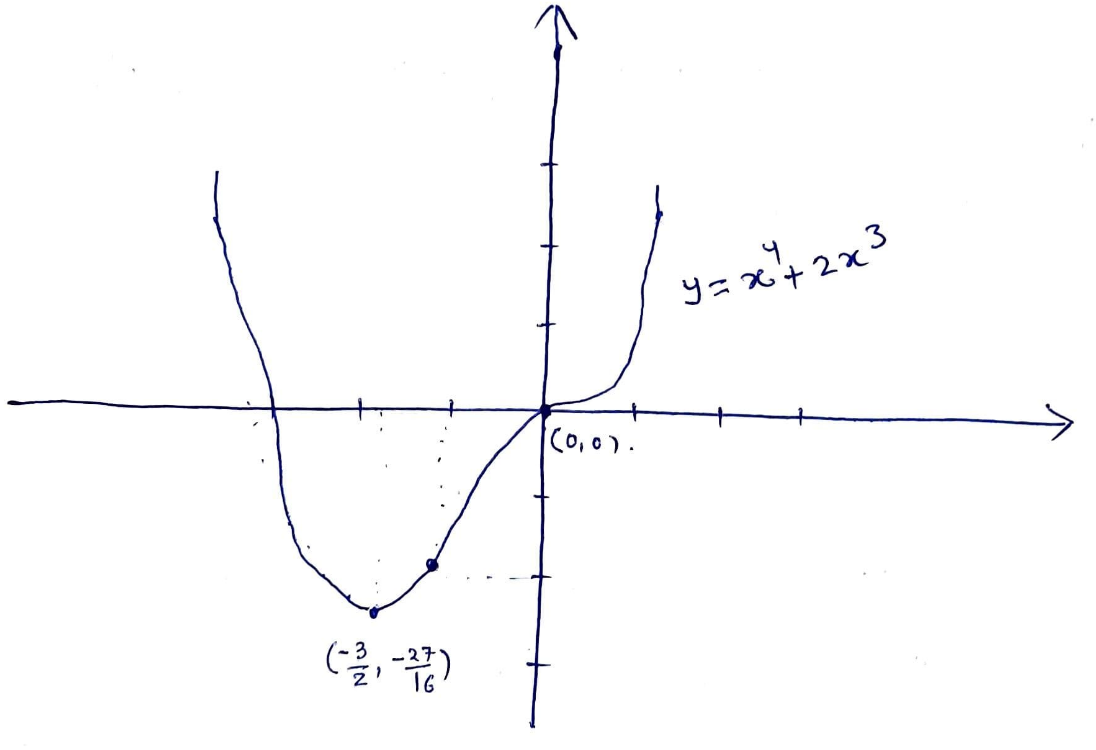
$\Rightarrow 12x(x+1) = 0$

$\Rightarrow x = 0$ or $x = -1$.

Intervals	$x < -1$.	$-1 < x < 0$	$x > 0$
Sing of y''	$+$	$-$	$+$
Behaviour of f	Concave up	Concave down	Concave up.

i.e. Curve is Concave up on $(-\infty, -1)$ and $(0, \infty)$ and Concave down on $(-1, 0)$.

at $x = -1$ and $x = 0$ these are the points of inflection.



Que 1 (ii)

given $y = x^4 - 2x^2$

then $y' = 4x^3 - 4x$

$= 4x(x^2 - 1)$

$= 4x \cdot (x-1)(x+1)$. —— (1)

and $y'' = 12x^2 - 4$

$= 12\left(x^2 - \frac{1}{3}\right)$

$= 12\left(x + \frac{1}{\sqrt{3}}\right)\left(x - \frac{1}{\sqrt{3}}\right)$. —— (2)

Thus the critical points of y occur only at the roots of y'.

i.e. $y' = 0$

$\Rightarrow 4x(x-1)(x+1) = 0$

$\Rightarrow x = 0, 1, \& -1$.

Critical points. $x = 0$ & $x = 1$ & $x = -1$.

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sing of y'	$-$	$+$	$-$	$+$
	Decreasing ↓	increasing ↑	↓	↑

Now at $x = 0$,

$y'' < 0$

, So there is a local maximum at $x = 0$.

at $x = 1$

$y'' > 0$

So there is a local minimum at $x = 1$.

at $x = -1$

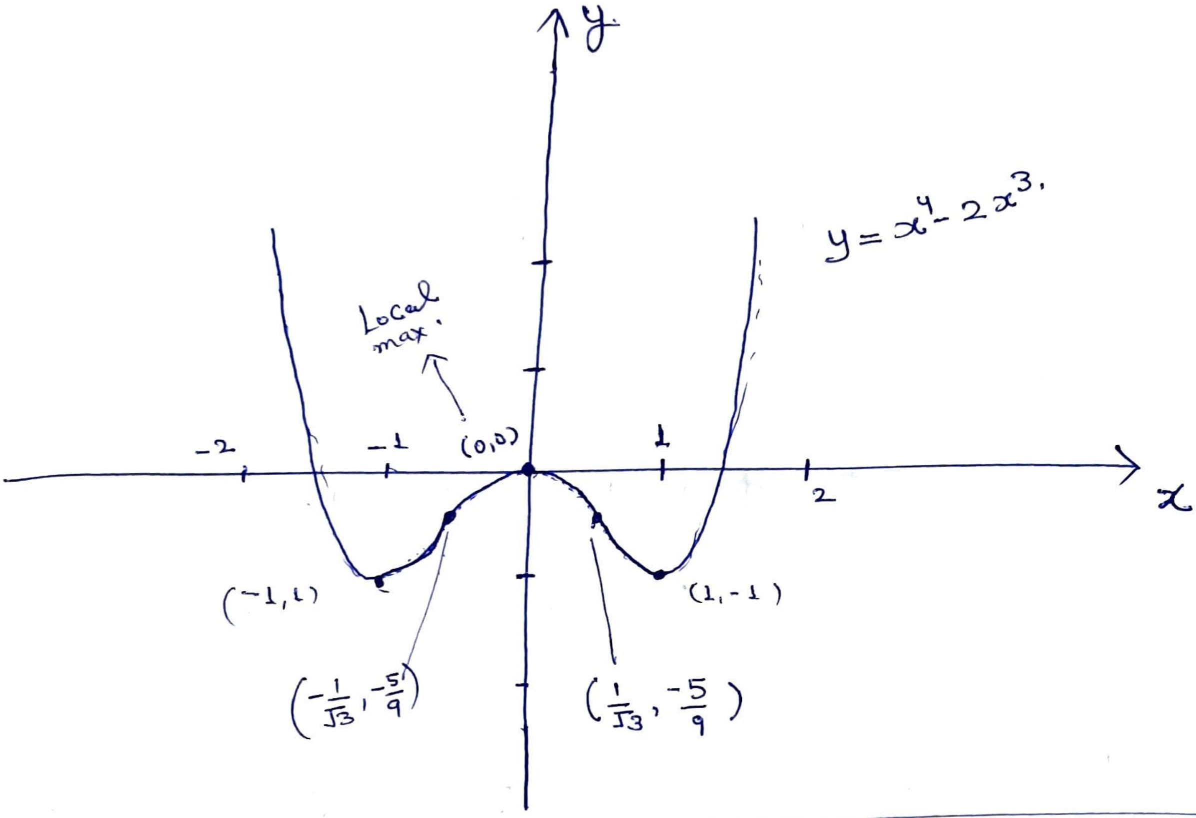
$y'' > 0$. So there is a local minimum at $x = -1$.

Now roots of y'' gives the inflection points.

i.e. $y'' = 0 \Rightarrow 12\left(x + \frac{1}{\sqrt{3}}\right)\left(x - \frac{1}{\sqrt{3}}\right) = 0$

$\Rightarrow x = \pm \frac{1}{\sqrt{3}}$ are the points of inflection.

Interval	$(-\infty, -\frac{1}{\sqrt{3}})$,	$(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$	$(\frac{1}{\sqrt{3}}, \infty)$
Sing of y''	$+$	$-$	$+$
	Concave up	Concave down	Concave up.



Sketch the graph of the following function.

Que. $y = x \cdot \left(\frac{x}{2} - 5\right)^4.$

Solⁿ: Try yourself.